Critical behavior for the onset of type-III intermittency observed in an electronic circuit

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Near the onset of type-III intermittent chaos observed in an autonomous electronic circuit, the critical exponent β for a relation $\langle\tau\rangle^{-1}\sim\varepsilon^{\beta}$ is experimentally determined, where $\langle\tau\rangle^{-1}$ is the inverse of average laminar length and ε is a parameter in the second return map, $I_{k+2}=(1+2\varepsilon)I_k+bI_k^3$. We obtain $\beta=0.55$, which is far from the Schuster theory value ($\beta=1$) and the recent experimental result ($\beta=0.85$) shown by Kahn, Mar, and Westervelt [Phys. Rev. B **45**, 8342 (1992)]. Our result is rather close to the theoretical value of 0.5 predicted by Kodama, Sato, and Honda [Phys. Lett. A **157**, 354 (1991)]. A similar result is also obtained for the critical behavior of the Lyapunov exponent.

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Recently, experiments on type-III intermittency have been performed in several nonlinear systems [1–4]. Results of these experiments can be well expressed by a one-dimensional map introduced by Pomeau and Manneville [5]. Good agreement between experiment and theory for the distribution of laminar length can also be obtained in these systems [1,3]. However, near the onset of intermittent chaos, analyses based on critical phenomena are quite few. Near the critical point a relation $\langle\tau\rangle^{-1}\sim\varepsilon^{\beta}$ holds for the inverse of average laminar length $\langle\tau\rangle^{-1}$, where ε is a parameter from the map by Pomeau and Manneville. It has been theoretically expressed by Schuster that the value of the critical exponent β is 1 [6]. Kahn, Mar, and Westenelt showed $\beta=0.85$ from their experimental data, and stated that their value is close to that of 1 obtained by Schuster [4]. However, Kodama, Sato, and Honda predicted $\beta=0.5$ by means of renormalization group techniques [7]. Which value of the critical exponent β is correct, 1 or 0.5? This question must be solved by experimentation. In this paper we experimentally present the critical exponent β from a type-III intermittent time series observed in an autonomous electronic circuit.

The electronic circuit used in this experiment is shown in Fig. 1. NR is a negative resistance which consists of an operational integrator and three resistors. Periodic oscillations with an amplitude of about 1 V and a frequency of about 5 kHz are spontaneously induced for the resistance R less than about 200 Ω. Such an oscillation reaches chaos by varying the resistance R, which is chosen as a control parameter. In this experiment voltages across the capacitor C_2 were measured with a 12-bit analog to digital converter. The sampling time was set to 20 μs in all measurements. The wave form data of about ten points a period can be taken for this sampling time. The data length was 16 (16 384 sampling points) or 64 kword (65 536 sampling points). We performed spline interpolation when needed.

A type-III intermittent time series observed at $R=789.2$ Ω is shown in Fig. 2. The growth of subharmonics can clearly be seen in this figure. In Fig. 3 we show a second return map determined from the time series in Fig. 2. The map function of the second return map can be written as

$$I_{k+2}=(1+2\varepsilon)I_k+aI_k^2+bI_k^3\ , \qquad (1)$$

where I_k denotes the kth peak value. Experimental data were fitted to Eq. (1) with $\varepsilon=0.125$, $a=0.006$, and $b=6.559$, where these parameters were determined by the least square method. It is a fact that $\varepsilon>0$ is essential for the occurrence of intermittency. The orbit is repulsive to the region in which the map intersects a line of $I_{k+2}=I_k$.

We consider the behavior of motion near the onset of the type-III intermittent chaos as a critical phenomenon. After the control parameter R increases beyond a critical

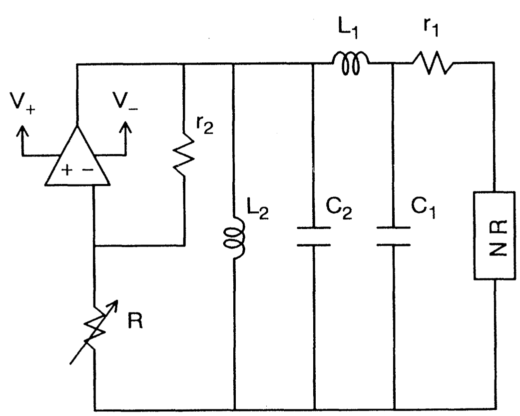

FIG. 1. Circuit diagram. NR denotes a resistor with a negative resistance. Values of elements are as follows: $r_1=64$ Ω, $r_2=300$ Ω, $C_1=0.022$ μF, $C_2=0.1$ μF, $L_1=32$ mH, and $L_2=22$ mH. The resistance R varies from 0 to 1978 Ω. The operational integrator is Model μA741.

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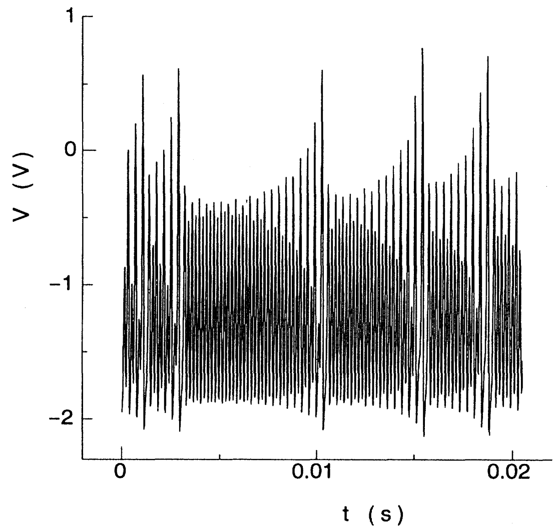

FIG. 2. Time series of type-III intermittency for the case of $R=789.2\ \Omega$.

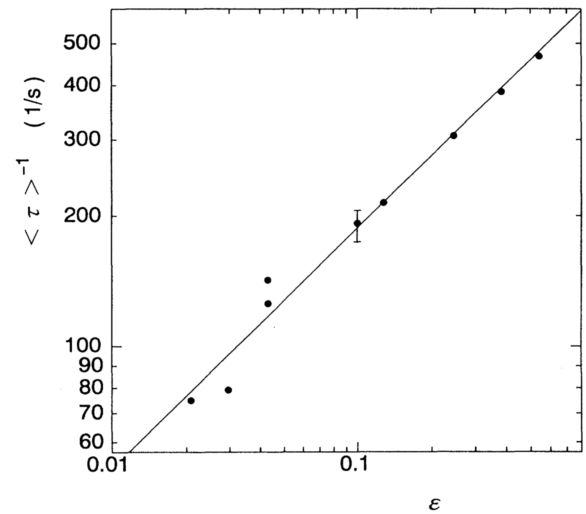

FIG. 4. ε dependence of the inverse of the average laminar length. The values of ε are estimated from the second return maps. A straight line is drawn by the least square method, and its slope is 0.55.

point R_c, the motion changes to a periodic oscillation. The average laminar length gradually increases as the control parameter approaches the critical point. That is, the inverse number of average laminar length, $\langle\tau\rangle^{-1}$, plays the same role as an order parameter in general critical phenomena. Displacement from the critical point is defined as

$$\epsilon=\frac{|R-R_c|}{R_c} . \tag{2}$$

It was difficult to determine the value of R_c from direct measurements. Therefore, we plotted $\langle\tau\rangle^{-1}$ as a function of R and determined by extrapolation the value of R_c from a point at which $\langle\tau\rangle^{-1}$ becomes 0. As a result, $R_c=832.9\ \Omega$ was obtained. The displacement ϵ must be proportional to the map parameter ε. We confirmed that the relation $\epsilon \propto \varepsilon$ holds very well by estimating the value ε from the second return map for various values of R. Theoretically, $\langle\tau\rangle^{-1}$ can be estimated from the distribution function $P(\tau)$. We obtain

$$\langle\tau\rangle=\frac{\int_{\tau_0}^{\infty}\tau P(\tau)d\tau}{\int_{\tau_0}^{\infty}P(\tau)d\tau}\sim\varepsilon^{-1/2} , \tag{3}$$

where

$$P(\tau)\sim\frac{\exp(-2\varepsilon\tau/\tau_0)}{[1-\exp(-4\varepsilon\tau/\tau_0)]^{3/2}} , \tag{4}$$

and hence the relation

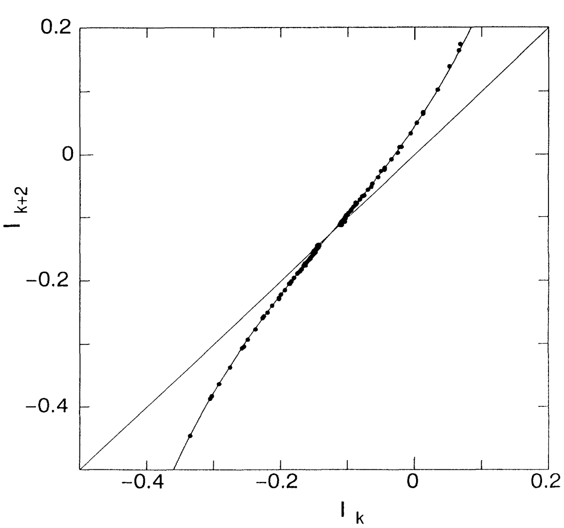

FIG. 3. Second return map. A curve denotes the map function $I_{k+2}=(1+2\varepsilon)I_k+aI_k^2+bI_k^3$ with $\varepsilon=0.125$, $a=0.006$, and $b=6.559$.

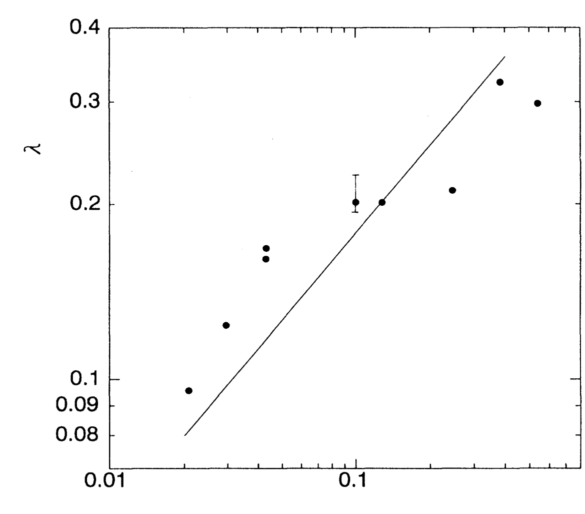

FIG. 5. ε dependence of the Lyapunov exponent. A straight line with slope 0.5 is also shown as an indication.

$$\langle \tau \rangle^{-1} \sim \varepsilon^{1/2} \quad (5)$$

is yielded. In order to verify the validity of relation (5), in Fig. 4 we show a log-log plot between $\langle \tau \rangle^{-1}$ and ε calculated from the experimental data. A straight line is drawn by use of the least square method. The slope of this line is 0.55. This implies that the following relation holds for $\langle \tau \rangle^{-1}$ and ε:

$$\langle \tau \rangle^{-1} \sim \varepsilon^{\beta} , \quad (6)$$

with $\beta \sim 0.55$.

In this experiment it was shown that the critical exponent β is nearly equal to 0.5. This fact does not agree with Schuster's theory and the experiment by Kahn, Mar, and Westervelt, but with the theory by Kodama, Sato, and Honda.

The ε dependence of the largest Lyapunov exponent λ should be similar to that of $\langle \tau \rangle^{-1}$, namely $\lambda \sim \langle \tau \rangle^{-1}$ [8]. We also calculated the Lyapunov exponents from the experimental time series by using the method of Wolf *et al.* [9]. The result is shown in Fig. 5, where we drew a line with slope 0.5 as an indication. These points do not fit well with the straight line in Fig. 5. However, if one takes into account the fact that calculation of the Lyapunov exponent from experimental data is not generally easy, it is considered that the relation $\lambda \sim \varepsilon^{0.5}$ can be found consistent with the relation for $\langle \tau \rangle^{-1}$.

In conclusion, the onset of type-III intermittent chaos was investigated from a standpoint of critical phenomena. When the inverse of average laminar length, $\langle \tau \rangle^{-1}$, is regarded as an order parameter, a relation $\langle \tau \rangle^{-1} \sim \varepsilon^{\beta}$ with $\beta = 0.55$ is obtained experimentally. The value of the critical exponent β (=0.55) is very close to the value 0.5 which is predicted by Kodama, Sato, and Honda. The ε dependence of the Lyapunov exponent λ can also be expressed as $\lambda \sim \varepsilon^{0.5}$.

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